Chapter 14

Oscillations

---

1 Marks Questions

1.The girl sitting on a swing stands up. What will be the effect on periodic time of swing?

Ans. The periodic time T is directly proportional to the square root of effective length of pendulum (l). When the girl starts up, the effective length of pendulum (i.e. Swing) decreases, so the Time period (T) also decreases.

2.At what distance from the mean position, is the kinetic energy in a simple harmonic oscillator equal to potential energy?

Ans. Let the displacement of particle executing S.H.M = Y

Amplitude of particle executing S.H.M = a

Mass of particle = m

Angular velocity = w

The kinetic energy = $\frac{1}{2}mw^2\left(a^2-y^2\right)$

$$Patential\ \text{energy} = \frac{1}{2}mw^2y^2$$

$$If\ \text{kinetic energy} = \text{Potential energy}$$

$$\frac{1}{2}mw^2\left(a^2-y^2\right) = \frac{1}{2}mw^2y^2$$

$$a^2-y^2=y^2$$

$a^2 = 2y^2$

$a = \sqrt{2y} \rightarrow$ Square root on both sides

$Y = \frac{a}{\sqrt{2}}$

3.The soldiers marching on a suspended bridge are advised to go out of steps. Why?

Ans. The soldiers marching on a suspended bridge are advised to go out of steps because in such a case the frequency of marching steps matches with natural frequency of the suspended bridge and hence resonance takes place, as a result amplitude of oscillation increases enormously which may lead to the collapsing of bridge.

4.Is the motion of a simple pendulum strictly simple harmonic?

Ans. It is not strictly simple harmonic because we make the assumption that Sinθ =θ, which is nearly valid only if θ is very small.

5.Can a simple pendulum experiment be done inside a satellite?

Ans. Since time period of a simple pendulum is :- $T = 2\pi\sqrt{\frac{1}{g}}$

Since, inside a satellite, effective value of 'g' = O

So, when g = O, T = α. Therefore, inside the satellite, the pendulum does not oscillate at all. So, it can not be preformed inside a satellite.

6.Give some practical examples of S. H. M?

Ans. Some practical examples of S. H. M. are :-

1) Motion of piston in a gas – filled cylinder.

2) Atoms vibrating in a crystal lattice.

3) Motion of helical spring.

7.What is the relation between uniform circular motion and S.H.N?

Ans. Uniform form circular motion can be thought of as two simple harmonic motion operating at right angle to each other.

8.What is the minimum condition for a system to execute S.H.N? .N? H i[p

Ans. The minimum condition for a body to posses S.H.N. is that it must have elasticity and inertia.

---

9. The piston in the cylinder head of a locomotive has a stroke (twice the amplitude) of 1.0 m. If the piston moves with simple harmonic motion with an angular frequency of 200 rad/min, what is its maximum speed?

Ans. Angular frequency of the piston, ω = 200 rad/ min.

Stroke = 1.0 m

Amplitude, $A = \frac{1.0}{2} = 0.5\, m$

The maximum speed (v_{max}) of the piston is give by the relation:

$$v_{max} = A\omega$$

$$= 200 \times 0.5 = 100\, m/\min$$

2 Marks Questions

1.A simple harmonic oscillator is represented by the equation : Y = 0.40 Sin (440t+0.61)

Y is in metres

t is in seconds

Find the values of 1) Amplitude 2) Angular frequency 3) Frequency of oscillation 4) Time period of oscillation, 5) Initial phase.

Ans. The given equation is:- Y = 0.40 Sin (440 t + 0.61)

Comparing it with equation of S.H.M. Y = a Sin (w t +$\varnothing_0$)

1) Amplitude = a = 0.40m

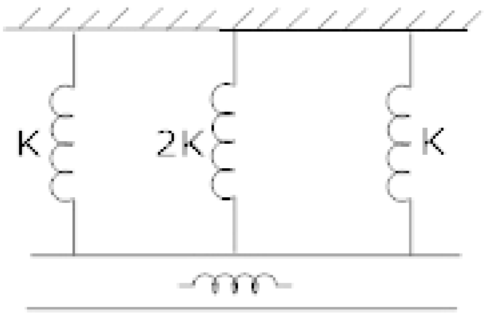
2) Angular frequency = w = 440Hz

3) Frequency of oscillation, $\upsilon = \frac{w}{2\pi} = \frac{440}{2\times\frac{22}{7}} = 70Hz$

4) Time period of oscillations, T = $\frac{1}{v} = \frac{1}{70} = 0.0143s$

5) Initial phase, $\varnothing$ = 0.61 rad.

2.The springs of spring factor k, 2k, k respectively are connected in parallel to a mass m. If the mass = 0.08kg m and k = 2N|m, then find the new time period?



Ans. Total spring constant, $K^1 = K_1 + K_2 + K_3$ (In parallel)

= K + 2K + K

= 4K

= 4 × 2 (k = 2 N | m)

= 8 N | m

Time period,

$$T = 2\pi\sqrt{\frac{m}{K^1}}$$

$$T = 2\pi\sqrt{\frac{m}{4K}}$$

$$T = 2\pi\sqrt{\frac{0.08}{4\times 2}}$$

$$T = 2\times\frac{22}{7}\times\sqrt{\frac{0.08}{8}}$$

$$T = 2\times\frac{22}{7}\times 0.1$$

$$T = 0.628 \text{ s}$$

3.The bob of a vibrating simple pendulum is made of ice. How will the period of swing will change when the ice starts melting?

Ans . The period of swing of simple pendulum will remain unchanged till the location of centre of gravity of the bob left after melting of the ice remains at the fixed position from the point of suspension. If centre of gravity of ice bob after melting is raised upwards, then effective length of pendulum decreases and hence time period of swing decreases. Similarly, if centre of gravity shifts downward, time period increases.

4.An 8 kg body performs S.H.M. of amplitude 30 cm. The restoring force is 60N, when the displacement is 30cm. Find: - a) Time period b) the acceleration c) potential and kinetic energy when the displacement is 12cm?

Ans. Here m = 8 kg

m = Mass, a = amplitude

a = 30cm = 0.30m

a) f = 60 N, Y = displacement = 0.30m

K = spring constant

Since, F = Ky

K = $\frac{F}{Y} = \frac{60}{0.30} = 200N|m$

As, Angular velocity = w = $\sqrt{\frac{k}{m}} = \sqrt{\frac{200}{8}} = 5|s$

Time period, T = $\frac{2\pi}{w} = \frac{2\times22}{7\times5} = \frac{44}{35} = 1.256s$

b) Y = displacement = 0.12m

Acceleration, A = w^2 y

A = $(5)^2 \times 0.12$

A = 3.0m |s^2

P.E. = Potential energy = $\frac{1}{2}ky^2 = \frac{1}{2}\times 200\times (0.12)^2$

$=\frac{1}{2}\times 200\times 144\times 10^{-4} = 1.44J$

Kinetic energy = K.E = $\frac{1}{2}k(a^2 - y^2)$

$K.E. = \frac{1}{2}\times 200\times (0.3^2 - 0.12^2$

= $\frac{1}{2}\times 200\times (0.09 - 0.0144)$

Kinetic energy = K. E. = 7.56J

5.A particle executing SH.M has a maximum displacement of 4 cm and its acceleration at a distance of 1 cm from its mean position is 3 cm/s^2. What will be its velocity when it is at a distance of 2cm from its mean position?

Ans. The acceleration of a particle executing S.H.M is –

A = w^2 Y

w = Angular frequency ; Y = Displacement

A = Acceleration

Given A = 3cm / s^2 ; Y = 1cm

So, 3 = w^2 × 1

w = $\sqrt{3} rad / s$

The velocity of a particle executing S.H.M is :-

$$V = w\sqrt{a^2 - y^2}$$

a = amplitude

$$V = \sqrt{3}\sqrt{(4)^2 - (2)^2}$$

$$V = \sqrt{3}\sqrt{16-4}$$

$$V = \sqrt{3} \times \sqrt{12}$$

$$V = \sqrt{3} \times 2 \times \sqrt{3}$$

$$V = 2 \times 3$$

$$V = 6cm/s$$

6.What is ratio of frequencies of the vertical oscillations when two springs of spring constant K are connected in series and then in parallel?

Ans . If two spring of spring constant K are connected in parallel, then effective resistance in parallel = K_P = K + K = 2K

Let f_P = frequency in parallel combination.

$$f_P = \frac{1}{2\pi}\sqrt{\frac{K_P}{M}}$$

Put the value of K_P

$$f_P = \frac{1}{2\pi}\sqrt{\frac{2K}{M}} \rightarrow (1)$$

In Series combination, effective spring constant for 2 sprigs of spring constant K is :-

$$\frac{1}{K_S} = \frac{1}{K} + \frac{1}{K}$$

$$\frac{1}{K_S} = \frac{K+K}{K \times K} = \frac{2K}{K^2}$$

$$\frac{1}{K_S} = \frac{K}{2} \; or \; K_S = \frac{K}{2}$$

Let f_S = frequency in series combination

$$f_S = \frac{1}{2\pi}\sqrt{\frac{K_S}{M}}$$

$$f_S = \frac{1}{2\pi}\sqrt{\frac{\frac{K}{2}}{M}}$$

$$f_S = \frac{1}{2\pi}\sqrt{\frac{K}{2M}} \rightarrow 2)$$

Divide equation 2) by 1)

$$\frac{f_S}{f_P} = \frac{\frac{1}{2\pi}\sqrt{\frac{K}{2M}}}{\frac{1}{2\pi}\sqrt{\frac{2K}{M}}}$$

$$\frac{f_S}{f_P} = \frac{1}{2\pi}\sqrt{\frac{K}{2M}} \frac{\times 2\pi \times \sqrt{M}}{\times\sqrt{2K}}$$

$$\frac{f_S}{f_P} = \sqrt{\frac{K \times M}{2M \times 2K}}$$

$$\frac{f_S}{f_P} = \sqrt{\frac{1}{4}}$$

$$\frac{f_S}{f_P} = \frac{1}{2}$$

$f_S : f_P = 1:2$

7.The kinetic energy of a particle executing S.H.M. is 16J when it is in its mean position. If the amplitude of oscillations is 25cm and the mass of the particle is 5.12kg. Calculate the time period of oscillations?

Ans.K. E. = Kinetic energy = 16J

Now, m = Mass = 5.12kg

W = Angular frequency

a = amplitude = 25cm or 0.25m

The Maximum value of K. E. is at mean position which is = $\frac{1}{2} m\, w^2 a^2$

So, $16 = \frac{1}{2} m w^2 a^2$

$16 X 2 = m w^2 a^2$

$32 = 5.12 X w^2 X (0.25)^2$

$32 = 5.12 X w^2 (625 X 10^{-4})$

$\frac{32}{5.12 X 625 X 10^{-4}} = w^2$

$\frac{32}{512 X 625 X 10^{-2-4}} = w^2$

$\frac{\cancel{32}\ 1}{\cancel{320000}_{1000} \times 10^{-6}} = w^2$

$10^{-4+6} = w^2$

$100 = w^2$

w = 10 rad/sec

Now, T = Time Period = $\frac{2\pi}{w} = \frac{2\pi}{10} = \frac{\pi}{5}$ sec.

8.The time period of a body suspended by a spring is T. What will be the new time period if the spring is cut into two equal parts and 1) the body is suspended by one part. 2) Suspended by both parts in parallel?

Ans. Since time period of oscillation, a body of mass 'm' suspended from a spring with force constant 'k' are:- $T = 2\pi\sqrt{\frac{m}{k}}$

1) On cutting the spring in two equal parts, the length of one part is halved and the force constant of each part will be doubled (2k). Therefore, the new time period is :→

$$T_1 = 2\pi\sqrt{\frac{m}{2K}}$$

If Initial Time - Period is $T = 2\pi\sqrt{\frac{m}{K}}$

So, $T^1 = \frac{T}{\sqrt{2}}$

2) If the body is suspended from both parts in parallel, then the effective force constant of parallel combination = 2k + 2k = 4k. Therefore, time period is:→

$$T_2 = 2\pi\sqrt{\frac{m}{4K}}$$

$$T_2 = \frac{T}{\sqrt{4}} \text{ or } \frac{T}{2}$$

9.A simple pendulum is executing Simple harmonic motion with a time T. If the length of the pendulum is increased by 21 %. Find the increase in its time period?

Ans. Now, time period of simple pendulum,

l = length of simple pendulum

g = acceleration due b gorily

$$T = 2\pi\sqrt{\frac{1}{g}}$$

If T_2 = Final time period

T_1 = Initial time period

So, $T_2 = 2\pi\sqrt{\frac{l_2}{g}}$

$$T_1 = 2\pi\sqrt{\frac{l_1}{g}}$$

Since 2π and g are constant, so;

$$T_2 = \sqrt{l_2} \rightarrow (1)$$

$$T_1 = \sqrt{l_1} \rightarrow (2)$$

Divide equation 2) by 1)

$$\frac{T_2}{T_1} = \sqrt{\frac{l_2}{l_1}}$$

If $l_1 = l$

$$l_2 = l_1 + l_1 \times \frac{21}{100}$$

$$l_2 = 1.21l$$

So, $\frac{T_2}{T_1} = \sqrt{\frac{1.21l}{l}}$

$\frac{T_2}{T_1} = 1.1 \quad T_2 = 1.1T_1$

Therefore percentage increase in time period = $\frac{T_2 - T_1}{T} \times 100\%$

$= \frac{1.1T - T}{T} \times 100\%$

$= \frac{0.1T}{T} \times 100\%$

$= 10\%$

10.A particle is executing S H M of amplitude 4 cm and T = 4 sec. find the time taken by it to move from positive extreme position to half of its amplitude?

Ans. If Y = displacement

t = time

a = amplitude

w = Angular frequency

Now, Y = a cos w t

Given Y = $\frac{a}{2}$

So, $\frac{a}{2} = a \text{ Cos w t}$

$\frac{1}{2} = \text{Cos w t}$

Now, T = Time Period, w = $\frac{2\pi}{T}$

$$\frac{1}{2} = Cos\frac{2\pi}{T}Xt \quad (T = 4\sec)$$

$$\frac{1}{2} = Cos\frac{2\pi}{4}Xt$$

Let WA is work done by spring A & kA = Spring Constant

WB is work done by spring B& kB = Spring Constant

$$\frac{WA}{WB} = \frac{kA}{kB} = \frac{1}{3} \therefore$$

$$WA : WB = 1 : 3$$

11.Two linear simple harmonic motions of equal amplitudes and angular frequency w and 2 w are impressed on a particle along axis X and Y respectively. If the initial phase difference between them is $\frac{\pi}{2}$, find the resultant path followed by the particle?

Ans. Let s = amplitude of each S.H.M.

Then give simple harmonic motions may be represented by: $\rightarrow$

W = Angular frequency

t = time

x = A Sin w t →1)

Now, $y = a \text{ Sin } \left(200t + \frac{\pi}{2}\right)$

x → Displacement along X – axis

y → Displacement along y – axis.

$$y = a \text{ Cos } 2 \text{ w t} \left[\because \text{ Sin } \left(\theta + \frac{\pi}{2} \right) = Cos\theta \right]$$

Now, Cos 2 θ = 1 – 2 Sin^2 θ

$$y = a\left(1 - 2Sin^2 w\ t\right)$$

For equation 1) Sin w t = $\frac{x}{a}$; Putting the value

Of Sin w t in equation 2)

$$y = a\left(1 - \frac{2x^2}{a^2}\right)$$

$$y = a - \frac{2x^2}{a}$$

$$y + \frac{2x^2}{a} = a$$

$$\frac{2x^2}{a} = a - y$$

$$x^2 = \frac{a}{2}(a - y)$$

$$x^2 = \frac{a^2}{2} - \frac{a}{2}y$$

12.The acceleration due to gravity on the surface of moon is 1.7 m/s^2. What is the time period of simple pendulum on moon if its time period on the earth is 3.5s?

Ans. If l = length of simple pendulum,

T = Time Period

g = Acceleration due to gravity.

Then, on earth ; $T = 2\pi\sqrt{\frac{l}{g}} \rightarrow (1)$

On Moon ; $T_1 = 2\pi\sqrt{\frac{l}{g_1}} \rightarrow 2)$

g_1 = Acceleration due to gravity on moon = 1.7 m | S^2

g = Acceleration due to gravity on earth = 9.8m | s^2

Dividing equation 2) by (1)

$$\frac{T_1}{T} = \sqrt{\frac{g}{g_1}}$$

$$\frac{T_1}{T} = \sqrt{\frac{9.8}{1.7}}$$

$$T_1 = T \times \sqrt{\frac{9.8}{1.7}}$$

$$T_1 = 3.5 \times \sqrt{\frac{9.8}{1.7}}$$

$$T_1 = 8.4s$$

---

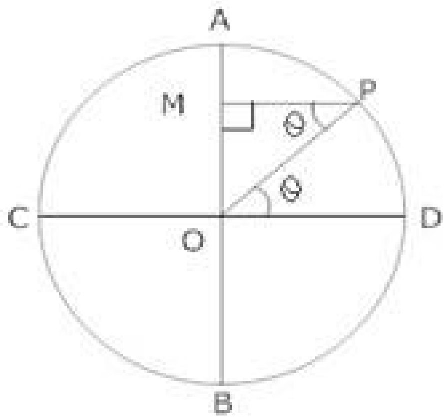
13. Using the correspondence of S. H. M. and uniform circular motion, find displacement, velocity, amplitude, time period and frequency of a particle executing SH.M?

Ans. If initially at t = 0

Particle is at D

Then at time = t

Particle is at point P



Then Drop a perpendicular From P on AB,

If the displacement OM = Y

Ratios of circle of reference = Amplitude = a

then In Δ O P M :→ Angle POD = Angle OPM (∵ Alternate Angles)

Sin $\theta = \frac{OM}{OP}$

$Sin\theta = \frac{y}{a}$

$Y = a \text{ Sin } \theta$

Now, w = Angular speed ⇒

T = time

Then $\theta = wt$

So, $Y = a$ Sin w t

2) Velocity, $V = \frac{dy}{dt}$

$$V = \frac{d}{dt}(\text{a Sin wt})$$

$$V = a\frac{d}{dt}(\text{Sin wt})$$

$$V = \text{a}[w.\text{Cos wt}]$$

$$V = \text{a w Cos wt}$$

Now, $\text{Sin}^2\,\theta + \text{Cos}^2\,\theta = 1$

$$\text{Cos}^2\ \theta = 1\text{-Sin}^2\ \theta$$

$$\text{Cos}\ \theta = \sqrt{1 - Sin^2\theta}$$

So, V = a w $\sqrt{1 - Sin^2 wt}$

Form equation of displacement :→ $\text{Sin wt} = \frac{y}{a}$

$$Sin^2 wt = \frac{y^2}{a^2}$$

So, V = a w $\sqrt{1 - \frac{y^2}{a^2}}$

$$V = aw\sqrt{\frac{a^2 - y^2}{a^2}}$$

$$V = w\sqrt{a^2 - y^2}$$

3) Acceleration :→ A = $\frac{dV}{dt}$

$$A = \frac{d}{dt}(\text{a w Cos w t})$$

$$A = aw\frac{d}{dt}(\text{Cos w t})$$

$$A = aw \times w(-Sin\ \text{w t})$$

$$A = -aw2\ \text{Sin w t}$$

Now, Y = Sin w t

$$A = -w^2 Y$$

The acceleration is proportional to negative of displacement is the characteristics of S. H. M.

4) $T = \frac{2\pi}{w}$ T=Time Period

5) $r = \frac{1}{T} = \frac{w}{2\pi}$ r = frequency.

14.A particle executing S.H.N. along a straight line has a velocity of um/s when its displacement from mean position is 3 m and 3 m / s when displacement is 4m. Find the time taken to travel 2.5 m from the positive extremity of its oscillation?

Ans. Velocity = v_1 = u m/s

then, displacement = 3m let y_1 = displacement = 3m

$$Now,\ v^2 = w^2\ (a^2 - y^2)$$

For first Case: →

$$v_1^2 = w^2\ (a^2 - y_1^2)$$

Putting the value of v_1 & y_1

$$16 = w^2(a^2 - 9) \rightarrow i)$$

For second case, v_2 = 3m / s and y_2 = displacement = 4m

$$So, v_2^2 = w^2\left(a^2 - y_2^2\right)$$

$$9 = w^2\left(a^2 - 16\right) \rightarrow 2)$$

Dividing eq^4 1) by 2)

$$\frac{16}{9} = \frac{\cancel{w^2}\left(a^2 - 9\right)}{\cancel{w^2}\left(a^2 - 16\right)}$$

$$16\left(a^2 - 16\right) = 9\left(a^2 - 9\right)$$

$$16\ a^2 - 256 = 9\ a^2\ - 81$$

$$16a^2 - 9a^2\ = 81 + 256$$

$$7a^2\ = 175$$

$$a^2\ = \frac{175}{7}$$

$$a\ = \sqrt{\frac{175}{7}}$$

$$a = \sqrt{25}$$

$$a = 5m$$

$$Now, v_1^2 = w^2\left(a_2 - y_1^2\right)$$

$$So, 16 = w^2 (25 - 9)$$

$$16 = w^2\ \ 16$$

$$w^2 = 1.$$

$w = 1$ rad/s

When the particle is 2.5m from the positive extreme position, its displacement from the mean position, y = 5 – 2.5 = 2.5m. Since the time is measured when the particle is at extreme position:→

$$y = a \text{ Cos } wt$$

$$2.5 = 5 \text{ Cos } (1 \times t)$$

$$\text{Cos } t = \frac{2.5}{5 \times 10}$$

$$Cos\ t = \frac{25}{50}$$

$$Cos\ t = \frac{1}{2}$$

$$Cos\ t = \frac{Cos \pi}{3}$$

$$t = \frac{\pi}{3}$$

$$t = \frac{3.14}{3}$$

$$t = 1.047s$$

15.Springs is spring constant K, 2K, 4K, K ----- are connected in series. A mass M Kg is attached to the lower end of the last spring and system is allowed to vibrate. What is the time period of oscillation?

Ans. For effective resistance of spring of individual spring constant k_1, k_2, ------ k_n

$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2} + - - - \frac{1}{k_{3n}}$$

$Now,\ k_1=k;\ k_2=2k;\ k_3=4k;$ ------

$$\frac{1}{k_{eff}}=\frac{1}{k}+\frac{1}{2k}+\frac{1}{4k}+\frac{1}{8k}+----$$

$$\frac{1}{k_{eff}}=\frac{1}{k}\left[1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+----\right]\because Sum \text{ of in finite G.P.} = \frac{a}{1-r}$$

$$\frac{1}{k_{eff}}=\frac{1}{k}\left[\frac{1}{1-\frac{1}{2}}\right]$$

$a = First$ Term

r = Common on Ratio

$$\frac{1}{k_{eff}}=\frac{1}{k}\left[\frac{1}{\frac{1}{2}}\right]$$

$$\frac{1}{k_{eff}}=\frac{2}{k}$$

$$k_{eff}=\frac{k}{2}$$

Since Time Perion, $T = 2\pi\sqrt{\frac{M}{k_{eff}}}$

$$T=2\pi\sqrt{\frac{M}{\frac{k}{2}}}$$

$$T=2\pi\sqrt{\frac{2M}{k}}$$

16.A particle is moving with SHN in a straight line. When the distance of the particle from mean position has values x1 and x2 the corresponding values of velocities are v1

and v2. Show that the time period of oscillation is given by:→

$$T = 2\pi\left[\frac{x_2^2 - x_1^2}{v_1^2 - v_2^2}\right]^{1/2}$$

Ans. If a = amplitude ; y = displacement; w = angular frequency

V = Velocity, then

$V^2 = w^2 (a^2 - y^2)$

For first case. $u_1^2 = w^2(a^2 - x_1^2)$ →1) (∵ velocity = u_1 Displacement = x_1)

For second case, $u_2^2 = w^2 (a^2 - x_2^2)$ →2) (velocity = u_2 Displacement = x_2)

Subtracting equation 2) from equation 1);

$$u_1^2 - u_2^2 = w^2\left(a^2 - x_1^2\right) - w^2\left(a^2 - x_2^2\right)$$

$$u_1^2 - u_2^2 = w^2a^2 - w^2x_1^2 - w^2a^2 + w^2x_2^2$$

$$u_1^2 - u_2^2 = w^2\left(x_2^2 - x_1^2\right)$$

$$Now,\ w^2 = \frac{u_1^2 - u_2^2}{x_2^2 - x_1^2}$$

$$So,\ w = \left[\frac{u_1^2 - u_2^2}{x_2^2 - x_1^2}\right]^{1/2}$$

$$So,\ \text{Time period, } T = \frac{2\pi}{w} = 2\pi\left[\frac{x_2^2 - x_1^2}{u_1^2 - u_2^2}\right]^{1/2}$$

---

17. A circular disc of mass 10 kg is suspended by a wire attached to its centre. The wire is twisted by rotating the disc and released. The period of torsional oscillations is found

to be 1.5 s. The radius of the disc is 15 cm. Determine the torsional spring constant of the wire. (Torsional spring constant α is defined by the relation *J* = -α θ , where *J* is the restoring couple and θ , the angle of twist).

Ans. Mass of the circular disc, *m* = 10 kg

Radius of the disc, *r* = 15 cm = 0.15 m

The torsional oscillations of the disc has a time period, *T* = 1.5 s

The moment of inertia of the disc is:

$$I = \frac{1}{2}mr^2$$

$$= \frac{1}{2} \times (10) \times (0.15)^2$$

= 0.1125 kg m^2

Time period, $T = 2\pi\sqrt{\frac{I}{a}}$

α is the torsional constant.

$$a = \frac{4\pi^2 I}{T^2}$$

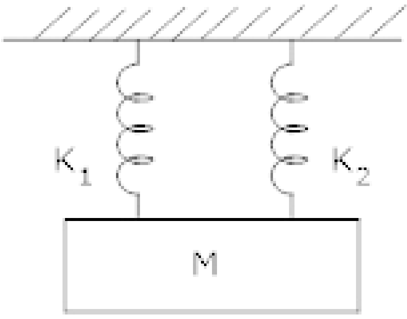
$$= \frac{4 \times (x)^2 \times 0.1125}{(1.5)^2}$$

= 1.972 Nm/rad

Hence, the torsional spring constant of the wire is 1.972 Nm rad^{-1}.

3 Marks Questions

1.A mass = m suspend separately from two springs of spring constant k_1 and k_2 gives time period t_1 and t_2 respectively. If the same mass is connected to both the springs as shown in figure. Calculate the time period 't' of the combined system?



Ans. If T = Time Period of simple pendulum

m = Mass

k = Spring constant

then, $T = 2\pi\sqrt{\frac{m}{k}}$

or $k = \frac{4\pi^2 m}{T^2}$

For first spring : $\rightarrow$ $k_1 = \frac{4\pi^2 m}{t_1^2}$ let $T = t_1$

For second spring : $\rightarrow k_2 = \frac{4\pi^2 m}{t_2^2}$ let $T = t_2$

When springs is connected in parallel, effective spring constant, k = k = $k_1 + k_2$

or $k = \frac{4\pi^2 m}{t_1^2} + \frac{4\pi^2 m}{t_2^2}$

If t = total time period

$$\frac{4\pi^2 m}{t^2} = \frac{4\pi^2 m}{t_1^2} + \frac{4\pi^2 m}{t_2^2}$$

$$\frac{1}{t^2} = \frac{1}{t_1^2} + \frac{1}{t_2^2}$$

Or $t^{-2} = t_1^{-2} + t_2^{-2}$

2.Show that the total energy of a body executing SHN is independent of time?

Ans. Let y = displacement at any time‘t’

a = amplitude

w = Angular frequency

v = velocity,

y = a Sin wt

So, $v = \frac{dy}{dt} = \frac{d}{dt}(\text{a Sin wt})$

$v = \text{a w Cos wt}$

Now, kinetic energy = K. E. = $\frac{1}{2} m \text{v}^2$

$$K.E. = \frac{1}{2} \text{ m w}^2\text{a}^2 \text{ Cos}^2 \text{ wt} \rightarrow 1)$$

Potential energy = $\frac{1}{2}ky^2$

$$P.E. = \frac{1}{2}ka^2 \text{ Sin}^2 \text{ wt } \rightarrow 2)$$

Adding equation 1) & 2)

Total energy = K.E.+ P.E

$$= \frac{1}{2}mw^2a^2 \text{ Cos}^2 \text{ wt} + \frac{1}{2} \text{ ka}^2 \text{ Sin}^2 \text{ wt}$$

$$\text{Since w} = \sqrt{\frac{k}{m}} \Rightarrow \text{w}^2\text{m} = \text{k}^2$$

$$\text{Total energy} = \frac{1}{2}mw^2a^2Cos^2wt + \frac{1}{2}ka^2Sin^2wt$$

$$= \frac{1}{2}ka^2 \text{ Cos}^2 \text{ wt} + \frac{1}{2} \text{ ka}^2 \text{ Sin}_2 \text{ wt}$$

$$= \frac{1}{2}\text{ka}^2 \left(Cos^2 \text{ wt+Sin}^2 \text{ wt}\right)$$

$$\text{Total energy} = \frac{1}{2}ka$$

Thus total mechanical energy is always constant is equal to $\frac{1}{2}ka^2$. The total energy is independent to time. The potential energy oscillates with time and has a maximum value of $\frac{ka^2}{2}$. Similarly K. E. oscillates with time and has a maximum value of $\frac{ka^2}{2}$. At any instant = constant = $\frac{ka^2}{2}$. The K. E or P.E. oscillates at double the frequency of S.H.M.

3.A particles moves such that its acceleration 'a' is given by a = -b x where x =

displacement from equilibrium position and b is a constant. Find the period of oscillation? [2]

Ans. Given that a = -b x, Since a $\propto$ x and is directed apposite to x, the particle do moves in S. H. M.

a = b x (in magnitude)

$$or \ \frac{x}{a} = \frac{1}{b}$$

$$\text{or } \frac{Displacement}{Accleration} = \frac{1}{b} \rightarrow 1)$$

$$\text{Time period} = T = 2\pi\sqrt{\frac{Displacement}{Accleration}}$$

Using equation 1)

$$T = 2\pi\sqrt{\frac{1}{b}}$$

4.A particle is S.H.N. is described by the displacement function: →

$$x = A \text{ Cos } (wt + \Phi); w = \frac{2\pi}{T}$$

If the initial (t = 0) position of the particle is 1 cm and its initial velocity is π cm | s, What are its amplitude and phase angle?

Ans. At $t = 0;\ x = 1cm;\ w = \pi|s$

$t = \text{time}$

$x = \text{Postition}$

$w = \text{Argular frequency}$

$\therefore\ x = A\ Cos\ (Wt + \phi)$

$1 = A\ Cos\ (\pi \times 0 + \phi)$

$1 = A\ Cos\ \phi$

$Now, v = \frac{dx}{dt} = \frac{d}{dt}(A\ Cos\ (wt + \phi))$

At $t = 0$ $v = \pi$ cm|s; $w = \pi$|s

$\not\pi = -A\not\pi\ Sin\ (\pi \times 0 + \phi)$

$\Rightarrow\ -1 = A\ Sin\ \phi\ \rightarrow 2)$

Squaring and adding 1) & 2)

$A2\ Cos^2\ \phi + A2\ Sin^2\phi = 1 + 1$

$A2\left(Cos^2\phi + Sin^2\phi\right) = 2$

$A^2 = 2$

$A = \sqrt{2}cm$

Dividing 2) by 1), we have :-

$\frac{\not A\ Sin\ \phi}{\not A\ Cos\ \phi} = -1$

$\tan\ \phi = -1$

$or\ \ \phi \equiv \tan^{-1}(-1)$

$\phi = \frac{3\pi}{4}$

5.Determine the time period of a simple pendulum of length = l when mass of bob = m Kg? [3]

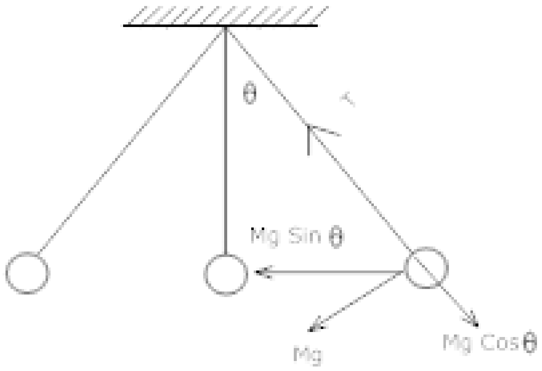
Ans. It consist of a heavy point mass body suspended by a weightless inextensible and perfectly flexible string from a rigid support which is free to oscillate.

The distance between point of suspension and point of oscillation is effective length of pendulum.

M = Mass of B ob

x = Displacement = OB

l = length of simple pendulum



Let the bob is displaced through a small angle θ the forces acting on it:-

1) weight = Mg acting vertically downwards.

2) Tension = T acting upwards.

Divide Mg into its components → Mg Cos θ & Mg Sin θ

T = Mg Cos θ

F = Mg Sin θ

- ve sign shows force is divested towards the ocean positions. If θ = Small,

Sin θ $\simeq \theta = \frac{Arc\ OB}{l} = \frac{x}{l}$

$F = -Mg\frac{x}{l}$

In S.H.M., vestoring fore, F = -mg θ F = -mg $\frac{x}{l} \rightarrow 1)$

Also, if k = spring constant

F = - k x

$\not{-}mg\ \frac{\not{x}}{l} = \not{-}k\ \not{x}\left(\text{equating } F = -mg\ \frac{x}{l}\right)$

$k = \frac{mg}{l}$

$T = 2\pi\sqrt{\frac{m}{k}}$

$= 2\pi\sqrt{\frac{\not{m} \times l}{\not{m}g}}$

$T = 2\pi\sqrt{\frac{l}{g}}$

i.e.1.) Time period depends on length of pendulum and 'g' of place where experiment is done.

2) T is independent of amplitude of vibration provided and it is small and also of the mass of bob.

---

6. Which of the following examples represent periodic motion?

(a) A swimmer completing one (return) trip from one bank of a river to the other and back.

(b) A freely suspended bar magnet displaced from its N-S direction and released.

(c) A hydrogen molecule rotating about its center of mass.

(d) An arrow released from a bow.

Ans.(b) and **(c)**

(a) The swimmer's motion is not periodic. The motion of the swimmer between the banks of a river is back and forth. However, it does not have a definite period. This is because the time taken by the swimmer during his back and forth journey may not be the same.

(b) The motion of a freely-suspended magnet, if displaced from its N-S direction and released, is periodic. This is because the magnet oscillates about its position with a definite period of time.

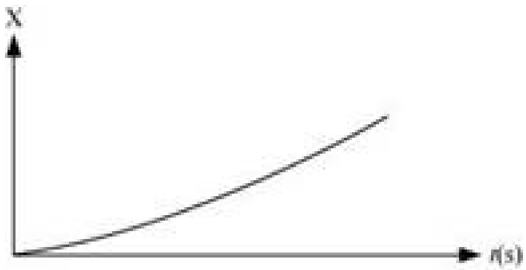
(c) When a hydrogen molecule rotates about its centre of mass, it comes to the same position again and again after an equal interval of time. Such motion is periodic.

(d) An arrow released from a bow moves only in the forward direction. It does not come backward. Hence, this motion is not a periodic.

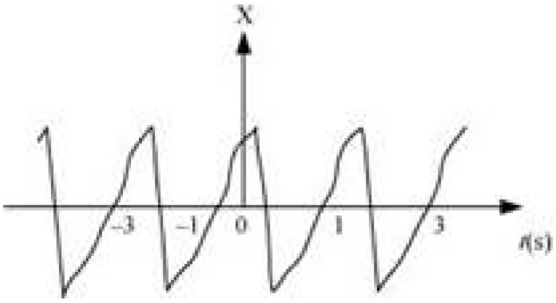
---

7. Figure 14.27 depicts four *x*-*t* plots for linear motion of a particle. Which of the plots represent periodic motion? What is the period of motion (in case of periodic motion)?

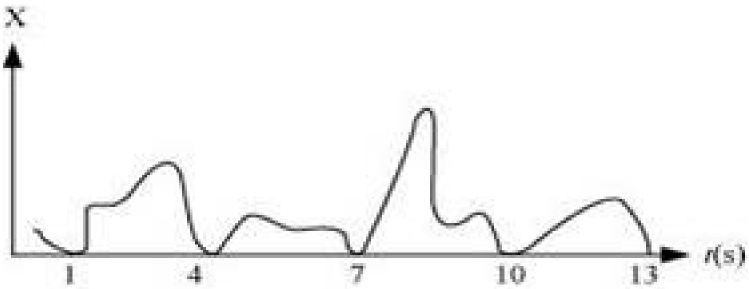
(a)



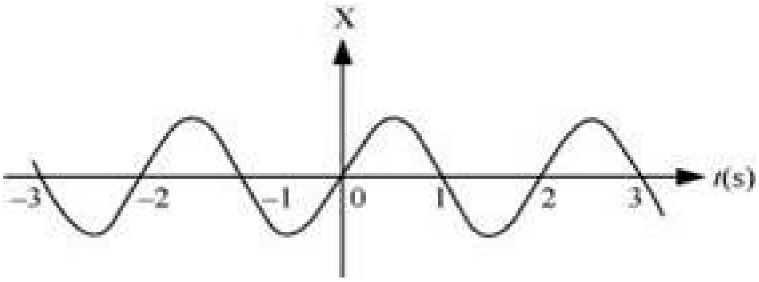
(b)



(c)



(d)



Ans.(b) and **(d)** are periodic

(a) It is not a periodic motion. This represents a unidirectional, linear uniform motion. There is no repetition of motion in this case.

(b) In this case, the motion of the particle repeats itself after 2 s. Hence, it is a periodic motion, having a period of 2 s.

(c) It is not a periodic motion. This is because the particle repeats the motion in one position only. For a periodic motion, the entire motion of the particle must be repeated in equal intervals of time.

(d) In this case, the motion of the particle repeats itself after 2 s. Hence, it is a periodic motion, having a period of 2 s.

8. Which of the following relationships between the acceleration a and the displacement x of a particle involve simple harmonic motion?

(a) $a = 0.7x$

(b) $a = -200x^2$

(c) $a = -10x$

(d) $a = 100x^3$

Ans.(c) A motion represents simple harmonic motion if it is governed by the force law:

$F = -kx$

$ma = -k$

$$\therefore a = -\frac{k}{m}x$$

Where,

F is the force

m is the mass (a constant for a body)

x is the displacement

a is the acceleration

k is a constant

Among the given equations, only equation $a = -10x$ is written in the above form with $\frac{k}{m} = 10$ Hence, this relation represents SHM.

---

9. The acceleration due to gravity on the surface of moon is 1.7 ms^{-2}. What is the time period of a simple pendulum on the surface of moon if its time period on the surface of earth is 3.5 s? (g on the surface of earth is 9.8 ms^{-2})

Ans. Acceleration due to gravity on the surface of moon, $g' = 1.7\ m\ s^{-2}$

Acceleration due to gravity on the surface of earth, g = 9.8 m s^{-2}

Time period of a simple pendulum on earth, T = 3.5 s

$$T = 2\pi\sqrt{\frac{l}{g}}$$

Where,

l is the length of the pendulum

$$\therefore l = \frac{T^2}{(2\pi)^2} \times g$$

$$= \frac{(3.5)^2}{4 \times (3.14)^2} \times 9.8\, m$$

The length of the pendulum remains constant.

On moon's surface, time period, $T' = 2\pi\sqrt{\frac{l}{g'}}$

$$= 2\pi\sqrt{\frac{\frac{(3.5)^2}{4 \times (3.14)^2} \times 9.8}{1.7}} = 8.4\, s$$

Hence, the time period of the simple pendulum on the surface of moon is 8.4 s.

---

10. A simple pendulum of length *l* and having a bob of mass *M* is suspended in a car. The car is moving on a circular track of radius *R* with a uniform speed *v*. If the pendulum makes small oscillations in a radial direction about its equilibrium position, what will be its time period?

Ans. The bob of the simple pendulum will experience the acceleration due to gravity and the centripetal acceleration provided by the circular motion of the car.

Acceleration due to gravity = g

Centripetal acceleration $= \frac{v^2}{R}$

Where,

v is the uniform speed of the car

R is the radius of the track

Effective acceleration (a_{eff}) is given as:

$$a_{eff} = \sqrt{g^2 + \left(\frac{v^2}{R}\right)^2}$$

Time period, $T = 2\pi\sqrt{\frac{l}{a_{eff}}}$

Where,l is the length of the pendulum

∴Time period, $T = 2\pi\sqrt{\frac{l}{\sqrt{g^2 + \frac{v^4}{R^2}}}}$

4 Marks Questions

1. Which of the following examples represent (nearly) simple harmonic motion and which represent periodic but not simple harmonic motion?

(a) the rotation of earth about its axis.

(b) motion of an oscillating mercury column in a U-tube.

(c) motion of a ball bearing inside a smooth curved bowl, when released from a point slightly above the lower most point.

(d) general vibrations of a polyatomic molecule about its equilibrium position.

Ans.(b) and **(c)** are SHMs

(a) and **(d)** are periodic, but not SHMs

(a) During its rotation about its axis, earth comes to the same position again and again in equal intervals of time. Hence, it is a periodic motion. However, this motion is not simple harmonic. This is because earth does not have a to and fro motion about its axis.

(b) An oscillating mercury column in a U-tube is simple harmonic. This is because the mercury moves to and fro on the same path, about the fixed position, with a certain period of time.

(c) The ball moves to and fro about the lowermost point of the bowl when released. Also, the ball comes back to its initial position in the same period of time, again and again. Hence, its motion is periodic as well as simple harmonic.

(d) A polyatomic molecule has many natural frequencies of oscillation. Its vibration is the superposition of individual simple harmonic motions of a number of different molecules. Hence, it is not simple harmonic, but periodic.

2. Which of the following functions of time represent (a) simple harmonic, (b) periodic but not simple harmonic, and (c) non-periodic motion? Give period for each case of periodic motion (ω is any positive constant):

(a) $\sin \omega t - \cos \omega t$

(b) $\sin^3 \omega t$

(c) 3 cos (π/4 - 2 ω t)

(d) cos ωt + cos 3ωt + cos 5ωt

(e) exp $(-\omega^2 t^2)$

(f) 1 + $\omega t + \omega^2 t^2$

Ans.(a) SHM

The given function is:

$\sin \omega t - \cos \omega t$

$$= \sqrt{2}\left[\frac{1}{\sqrt{2}} \sin \omega t - \frac{1}{\sqrt{2}} \cos \omega t\right]$$

$$= \sqrt{2}\left[\sin \omega t \times \cos \frac{\pi}{4} - \cos \omega t \times \sin \frac{\pi}{4}\right]$$

$$= \sqrt{2} \sin\left[\omega t - \frac{\pi}{4}\right]$$

This function represents SHM as it can be written in the form: $a \sin(\omega t + \phi)$

Its period is: $\frac{2\pi}{\omega}$

(b) Periodic, but not SHM

The given function is:

$sin^3 \omega t = 14[3 sin \omega t - sin^3 \omega$

---

3. A spring balance has a scale that reads from 0 to 50 kg. The length of the scale is 20 cm. A body suspended from this balance, when displaced and released, oscillates with a period of 0.6 s. What is the weight of the body?

Ans. Maximum mass that the scale can read, M = 50 kg

Maximum displacement of the spring = Length of the scale, l = 20 cm = 0.2 m

Time period, T = 0.6 s

Maximum force exerted on the spring, $F = Mg$

Where,

g = acceleration due to gravity = 9.8 m/ s^2

$F = 50 \times 9.8 = 490$

$\therefore$Spring constant, $k = \frac{F}{l} = \frac{490}{0.2} = 2450\, Nm^{-1}$

Mass m, is suspended from the balance.

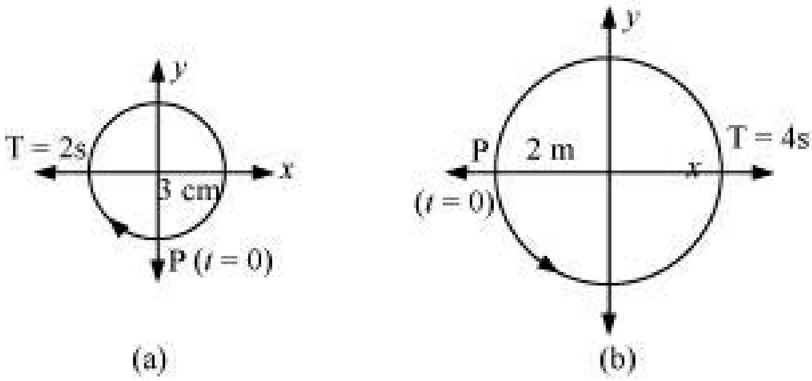
Time period, $T = 2\pi\sqrt{\frac{m}{k}}$

$\therefore = \left(\frac{T}{2\pi}\right)^2 \times k = \left(\frac{0.6}{2\times 3.14}\right)^2 \times 2450 = 22.36\, kg$

$\therefore$Weight of the body = mg = 22.36 $\times$ 9.8 = 219.167 N

Hence, the weight of the body is about 219 N.

4. Figures 14.29 correspond to two circular motions. The radius of the circle, the period of revolution, the initial position, and the sense of revolution (i.e. clockwise or anti-clockwise) are indicated on each figure.



Obtain the corresponding simple harmonic motions of the *x*-projection of the radius vector of the revolving particle P, in each case.

Ans.(a) Time period, $T = 2$ s

Amplitude, $A = 3$ cm

At time, $t = 0$, the radius vector OP makes an angle $\frac{\pi}{2}$ with the positive *x*-axis, i.e., phase angle $\phi = +\frac{\pi}{2}$

Therefore, the equation of simple harmonic motion for the*x*-projection of OP, at time *t*, is given by the displacement equation:

$$x = A\cos\left[\frac{2\pi t}{T} + \phi\right]$$

$$= 3\cos\left(\frac{2\pi t}{2} + \frac{\pi}{2}\right) = -3\sin\left(\frac{2\pi t}{2}\right)$$

$$\therefore x = -3\sin \pi t \, cm$$

(b) Time period, $T = 4$ s

Amplitude, $a = 2$ m

At time t = 0, OP makes an angle π with the x-axis, in the anticlockwise direction. Hence, phase angle, $\Phi = +\pi$

Therefore, the equation of simple harmonic motion for the x-projection of OP, at time t, is given as:

$$x = a\cos\left(\frac{2\pi t}{T}+\phi\right) = 2\cos\left(\frac{2\pi t}{4}+\pi\right)$$

$$\therefore x = -2\cos\left(\frac{\pi}{2}t\right) m$$

---

5. Cylindrical piece of cork of density of base area *A* and height *h* floats in a liquid of density ρ_l. The cork is depressed slightly and then released. Show that the cork oscillates up and down simple harmonically with a period

$$T = 2\pi\sqrt{\frac{h\rho}{\rho_l g}}$$

Where ω is the density of cork. (Ignore damping due to viscosity of the liquid).

Ans. Base area of the cork = A

Height of the cork = h

Density of the liquid = ρ_l

Density of the cork = ω

In equilibrium:

Weight of the cork = Weight of the liquid displaced by the floating cork

Let the cork be depressed slightly by x. As a result, some extra water of a certain volume is displaced. Hence, an extra up-thrust acts upward and provides the restoring force to the cork.

Up-thrust = Restoring force, F = Weight of the extra water displaced

F = –(Volume × Density × g)

Volume = Area × Distance through which the cork is depressed

Volume = Ax

$\therefore F = -A \times \rho_l g$ ... (i)

According to the force law:

$F = kx$

$$k = \frac{F}{x}$$

Where, k is a constant

$$k = \frac{F}{x} = -A\rho_1 g \quad ....\text{(ii)}$$

The time period of the oscillations of the cork:

$$T = 2\pi\sqrt{\frac{m}{k}} \quad ....\text{(iii)}$$

Where,

m = Mass of the cork

= Volume of the cork × Density

= Base area of the cork × Height of the cork × Density of the cork

= $Ah\rho$

Hence, the expression for the time period becomes:

$$T = 2\pi\sqrt{\frac{Ah\rho}{A\rho_l g}} = 2\pi\sqrt{\frac{h\rho}{\rho_l g}}$$

6. A mass attached to a spring is free to oscillate, with angular velocity ω , in a horizontal plane without friction or damping. It is pulled to a distance *x*0and pushed towards the centre with a velocity *v*0at time *t* = 0. Determine the amplitude of the resulting oscillations in terms of the parameters Ã”°, *x*0and *v*0. [Hint: Start with the equation *x* = *a* cos ($\omega t+\theta_{,}$) and note that the initial velocity is negative.]

Ans. The displacement equation for an oscillating mass is given by:

x= $A\cos\left(\omega t+\theta\right)$

Where,

A is the amplitude

X is the displacement

θ is the phase constant

Velocity, $v=\frac{dx}{dt}=-A\omega\sin\left(\omega t+\theta\right)$

At *t*= 0, $x=x_0$

$x_0=Acos\theta=x_0$ ... (*i*)

And, $\frac{dx}{dt}=-v_0=A\omega\sin\theta$

$A\sin\theta=\frac{v_0}{\omega}$ ... (*ii*)

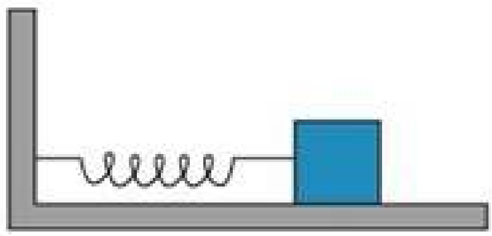
Squaring and adding equations (*i*) and (*ii*), we get:

$$A^2\left(\cos^2\theta+\sin^2\theta\right)=x_0^2+\left(\frac{v_0^2}{\omega^2}\right)$$

$$\therefore A=\sqrt{x_0^2+\left(\frac{v_0}{\omega}\right)^2}$$

Hence, the amplitude of the resulting oscillation is $\sqrt{x_0^2+\left(\frac{v_0}{\omega}\right)^2}$.

7. A spring having with a spring constant 1200 N m^{-1} is mounted on a horizontal table as shown in Fig. A mass of 3 kg is attached to the free end of the spring. The mass is then pulled sideways to a distance of 2.0 cm and released.



Determine (i) the frequency of oscillations, (ii) maximum acceleration of the mass, and (iii) the maximum speed of the mass.

Ans. Spring constant, k = 1200 N m^{-1}

Mass, m = 3 kg

Displacement, A = 2.0 cm = 0.02 cm

(i) Frequency of oscillation v, is given by the relation:

$$v=\frac{1}{T}=\frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

Where, T is the time period

$$\therefore v=\frac{1}{2\times 3.14}\sqrt{\frac{1200}{3}}=3.18\,m/s$$

Hence, the frequency of oscillations is 3.18 cycles per second.

(ii) Maximum acceleration (a) is given by the relation:

a = $\omega^2 A$

Where,

ω = Angular frequency = $\sqrt{\frac{k}{m}}$

A = Maximum displacement

$$\therefore a = \frac{k}{m}A = \frac{1200 \times 0.02}{3} = 8ms^{-2}$$

Hence, the maximum acceleration of the mass is 8.0 m/s^2.

(iii) Maximum velocity, $v_{max} = A\omega$

$$= A\sqrt{\frac{k}{m}} = 0.02 \times \sqrt{\frac{1200}{3}} = 0.4\,m/s$$

Hence, the maximum velocity of the mass is 0.4 m/s.

---

8. Answer the following questions:

(a) Time period of a particle in SHM depends on the force constant *k* and mass *m* of the particle:

$T = 2\pi\sqrt{\frac{m}{k}}$. A simple pendulum executes SHM approximately. Why then is the time period of a pendulum independent of the mass of the pendulum?

(b) The motion of a simple pendulum is approximately simple harmonic for small angle oscillations. For larger angles of oscillation, a more involved analysis shows that *T* is greater than $2\pi\sqrt{\frac{l}{g}}$. Think of a qualitative argument to appreciate this result.

(c) A man with a wristwatch on his hand falls from the top of a tower. Does the watch give correct time during the free fall?

(d) What is the frequency of oscillation of a simple pendulum mounted in a cabin that is

freely falling under gravity?

Ans.(a) The time period of a simple pendulum, $T = 2\pi\sqrt{\frac{m}{k}}$

For a simple pendulum, *k* is expressed in terms of mass *m*, as:

$k \propto m$

$\frac{m}{k}$ = Constant

Hence, the time period *T*, of a simple pendulum is independent of the mass of the bob.

(b) In the case of a simple pendulum, the restoring force acting on the bob of the pendulum is given as:

$F = -mg \sin\theta$

Where,

F = Restoring force

m = Mass of the bob

g = Acceleration due to gravity

θ = Angle of displacement

For small θ, $\sin\theta \simeq \theta$

For large θ, $\sin\theta$ is greater than θ.

This decreases the effective value of *g*.

Hence, the time period increases as:

T= $2\pi\sqrt{\frac{l}{g}}$

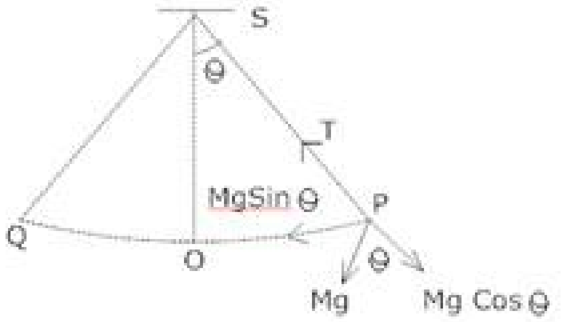
Where, l is the length of the simple pendulum

(c) The time shown by the wristwatch of a man falling from the top of a tower is not affected by the fall. Since a wristwatch does not work on the principle of a simple pendulum, it is not affected by the acceleration due to gravity during free fall. Its working depends on spring action.

(d) When a simple pendulum mounted in a cabin falls freely under gravity, its acceleration is zero. Hence the frequency of oscillation of this simple pendulum is zero.

5 Marks Questions

1.What is Simple pendulum? Find an expression for the time period and frequency of a simple pendulum?



Ans.A simple pendulum is the most common example of the body executing S.H.M, it consist of heavy point mass body suspended by a weightless inextensible and perfectly flexible string from a rigid support, which is free to oscillate.

Let m = mass of bob

l = length of pendulum

Let O is the equilibrium position, OP = X

Let θ = small angle through which the bob is displaced.

The forces acting on the bob are:-

1) The weight = M g acting vertically downwards.

2) The tension = T in string acting along Ps.

Resolving Mg into 2 components as Mg Cos θ and Mg Sin θ,

Now, T = Mg Cos θ

Restoring force F = - Mg Sin θ

- ve sign shows force is directed towards mean position.

Let θ = Small, so Sin θ ≈ θ = $\frac{Arc(op)}{l} = \frac{x}{l}$

Hence F = - mg θ

F = - mg $\frac{x}{l} \rightarrow 3)$

Now, In S.H.M, F = k x →4) k = Spring constant

Equating equation3) & 4) for F

- k x = - m g $\frac{x}{l}$

Spring factor = k = $\frac{mg}{l}$

Inertia factor = Mass of bob = m

Now, Time period = T

$= 2\pi\sqrt{\frac{Inertia\ factor}{Spring\ factor}}$

$= 2\pi\sqrt{\frac{m}{mg/l}}$

$T = 2\pi\sqrt{\frac{l}{g}}$

---

2. A particle is in linear simple harmonic motion between two points, A and B, 10 cm apart. Take the direction from A to B as the positive direction and give the signs of velocity, acceleration and force on the particle when it is

(a) at the end A,

(b) at the end B,

(c) at the mid-point of AB going towards A,

(d) at 2 cm away from B going towards A,

(e) at 3 cm away from A going towards B, and

(f) at 4 cm away from B going towards A.

Ans.(a) Zero, Positive, Positive

(b) Zero, Negative, Negative

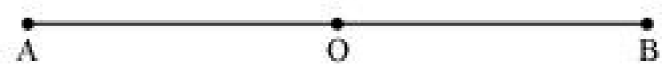
(c) Negative, Zero, Zero

(d) Negative, Negative, Negative

(e) Zero, Positive, Positive

(f) Negative, Negative, Negative

Explanation:
The given situation is shown in the following figure. Points A and B are the two end points, with AB = 10 cm. O is the midpoint of the path.



A particle is in linear simple harmonic motion between the end points

(a) At the extreme point A, the particle is at rest momentarily. Hence, its velocity is zero at this point.

Its acceleration is positive as it is directed along AO.

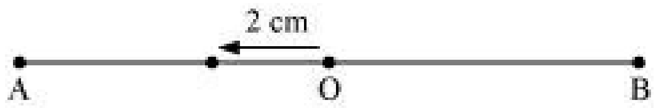
Force is also positive in this case as the particle is directed rightward.

(b) At the extreme point B, the particle is at rest momentarily. Hence, its velocity is zero at this point.

Its acceleration is negative as it is directed along B.

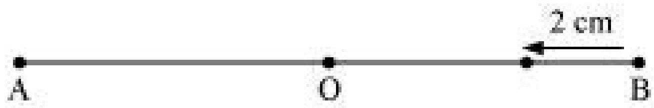
Force is also negative in this case as the particle is directed leftward.

(c)



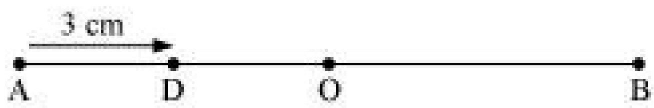
The particle is executing a simple harmonic motion. O is the mean position of the particle. Its velocity at the mean position O is the maximum. The value for velocity is negative as the particle is directed leftward. The acceleration and force of a particle executing SHM is zero at the mean position.

(d)



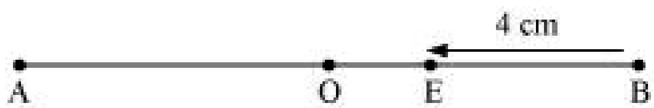
The particle is moving toward point O from the end B. This direction of motion is opposite to the conventional positive direction, which is from A to B. Hence, the particle's velocity and acceleration, and the force on it are all negative.

(e)



The particle is moving toward point O from the end A. This direction of motion is from A to B, which is the conventional positive direction. Hence, the values for velocity, acceleration, and force are all positive.

(f)



This case is similar to the one given in **(d)**.

3. The motion of a particle executing simple harmonic motion is described by the displacement function,

x (t) = A cos (ω t + ω).

If the initial (t = 0) position of the particle is 1 cm and its initial velocity is ω cm/s, what are its amplitude and initial phase angle? The angular frequency of the particle is π s-1. If instead of the cosine function, we choose the sine function to describe the SHM: x = B *sin* (ω t + α), what are the amplitude and initial phase of the particle with the above initial conditions.

Ans. Initially, at t = 0:

Displacement, x = 1 cm

Initial velocity, v = ω cm/sec.

Angular frequency, ω = π rad/ s^{-1}

It is given that:

$$x(x) = A\cos(\omega t + \phi)$$

$$1 = A\cos(\omega \times 0 + \phi) = A\cos\phi$$

$A\cos\phi = 1$ .....(i)

Velocity, $v = \dfrac{dx}{dt}$

$$\omega = -A\omega\sin(\omega t + \phi)$$

$$1 = A\sin(\omega \times 0 + \phi) = A\sin\phi$$

$A\sin\phi = -1$ ....(ii)

Squaring and adding equations (i) and (ii), we get:

$$A^2\left(\sin^2\phi+\cos^2\phi\right)=1+1$$

$$A^2=2$$

$$\therefore A=\sqrt{2}\ cm$$

Dividing equation (*ii*) by equation (*i*), we get:

$$\tan\phi=-1$$

$$\therefore \phi=\frac{3\pi}{4},\frac{7\pi}{4},\ldots\ldots\ldots$$

SHM is given as:

$$x=B\sin\left(\omega t+a\right)$$

Putting the given values in this equation, we get:

$$1=B\sin\left[\omega\times 0+a\right]$$

$B\sin a=1$ ....(iii)

Velocity, $v=\omega B\cos\left(\omega t+a\right)$

Substituting the given values, we get:

$$\pi=\pi B\sin a$$

$B\sin a=1$ ....(iv)

Squaring and adding equations (*iii*) and (*iv*), we get:

$$B^2\left[\sin^2 a+\cos^2 a\right]=1+1$$

$$B^2=2$$

$$\therefore B=\sqrt{2}\ cm$$

Dividing equation (*iii*) by equation (*iv*), we get:

$$\frac{B \sin a}{B \cos a} = \frac{1}{1}$$

$$\tan a = 1 = \tan \frac{\pi}{4}$$

$$a \frac{\pi}{4}, \frac{5\pi}{4}, \ldots\ldots\ldots$$

---

4. In Exercise 14.9, let us take the position of mass when the spring is unstreched as *x* = 0, and the direction from left to right as the positive direction of *x*-axis. Give *x* as a function of time t for the oscillating mass if at the moment we start the stopwatch (*t* = 0), the mass is

(a) at the mean position,

(b) at the maximum stretched position, and

(c) at the maximum compressed position.

In what way do these functions for SHM differ from each other, in frequency, in amplitude or the initial phase?

Ans.(a) $x = 2\sin 20t$

(b) $x = 2\cos 20t$

(c) $x = -2\cos 20t$

The functions have the same frequency and amplitude, but different initial phases.

Distance travelled by the mass sideways, $A = 2.0$ cm

Force constant of the spring, $k = 1200$ N m^{-1}

Mass, $m = 3$ kg

Angular frequency of oscillation:

$$\omega = \sqrt{\frac{k}{m}}$$

$$= \sqrt{\frac{1200}{3}} = \sqrt{400} = 20\, rad\, s^{-1}$$

(a) When the mass is at the mean position, initial phase is 0.

Displacement, $x = A\sin\ \omega t$

$= 2\sin 20t$

(b) At the maximum stretched position, the mass is toward the extreme right. Hence, the initial phase is $\frac{\pi}{2}$.

Displacement, $x = A\sin\left(\omega t + \frac{\pi}{2}\right)$

$$= 2\sin\left(20t + \frac{\pi}{2}\right)$$

$= 2\cos 20t$

(c) At the maximum compressed position, the mass is toward the extreme left. Hence, the initial phase is $\frac{3\pi}{2}$.

Displacement, $x = A\sin\left(\omega t + \frac{3\pi}{2}\right)$

$$= 2\sin\left(20t + \frac{3\pi}{2}\right)$$

$= -2\cos 20t$

The functions have the same frequency $\left(\frac{20}{2\pi} Hz\right)$ and amplitude (2 cm), but different initial phases $\left(0, \frac{\pi}{2}, \frac{3\pi}{2}\right)$.

---

5. Plot the corresponding reference circle for each of the following simple harmonic motions. Indicate the initial (*t* = 0) position of the particle, the radius of the circle, and the angular speed of the rotating particle. For simplicity, the sense of rotation may be fixed to be anticlockwise in every case: (*x* is in cm and *t* is in *s*).

(a) $x = -2 \sin (3t + \pi/3)$

(b) $x = \cos (\pi/6 - t)$

(c) $x = 3 \sin (2\pi t + \pi/4)$

(d) $x = 2 \cos \pi t$

Ans.(a) $x = -2\sin\left(3t + \frac{\pi}{3}\right) = +2\cos\left(3t + \frac{\pi}{3} + \frac{\pi}{2}\right)$

$= 2\cos\left(3t + \frac{5\pi}{6}\right)$

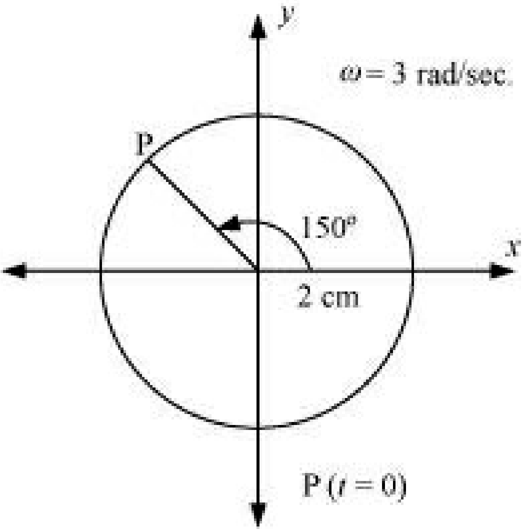
If this equation is compared with the standard SHM equation $x = A\cos\left(\frac{2\pi}{T}t + \phi\right)$, then we get:

Amplitude, A = 2 cm

Phase angle, $\phi = \frac{5\pi}{6} = 150°$

Angular velocity, $\omega = \frac{2\pi}{T} = 3\ rad / \sec$.

The motion of the particle can be plotted as shown in the following figure.



(b) $x = \cos\left(\frac{\pi}{6} - t\right) = \cos\left(t - \frac{\pi}{6}\right)$

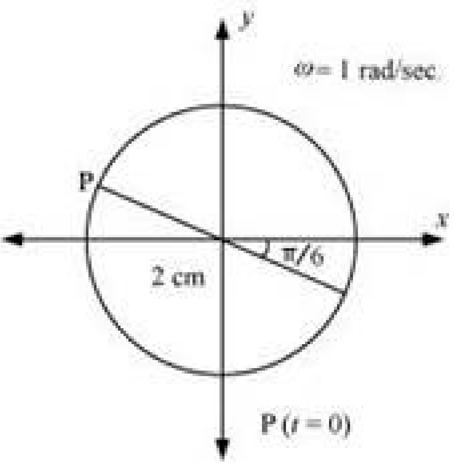
If this equation is compared with the standard SHM equation $x = A\cos\left(\frac{2\pi}{T}t + \phi\right)$, then we get:

Amplitude, A=2

Phase angle, $\phi = \frac{\pi}{6} = 30°$

Angular velocity, $\omega = \frac{2\pi}{T} = 1 rad / \sec.$

The motion of the particle can be plotted as shown in the following figure.



(c) $x = 3\sin\left(2\pi t + \frac{\pi}{4}\right)$

$$= -3\cos\left[\left(2\pi t + \frac{\pi}{4}\right) + \frac{\pi}{2}\right] = -3\cos\left(2\pi t + \frac{3\pi}{4}\right)$$

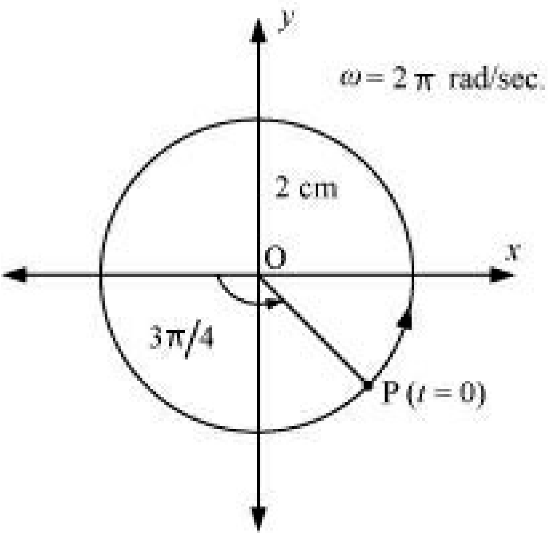
If this equation is compared with the standard SHM equation $x = A\cos\left(\frac{2\pi}{T}t + \phi\right)$, then we get:

Amplitude, A = 3 cm

Phase angle, $\phi = \frac{3\pi}{4}$ = 135°

Angular velocity, $\omega = \frac{2\pi}{T} = 2\pi\, rad/\sec.$

The motion of the particle can be plotted as shown in the following figure.



(d) $x = 2\cos \pi t$

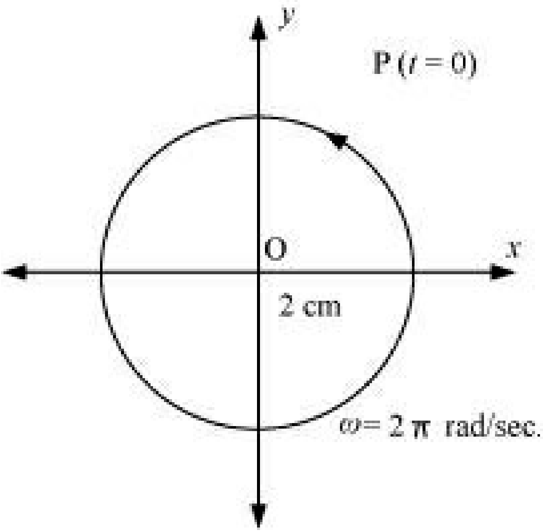
If this equation is compared with the standard SHM equation $x = A\cos\left(\frac{2\pi}{T}t + \phi\right)$, then we get:

Amplitude, A = 2 cm

Phase angle, ϕ = 0

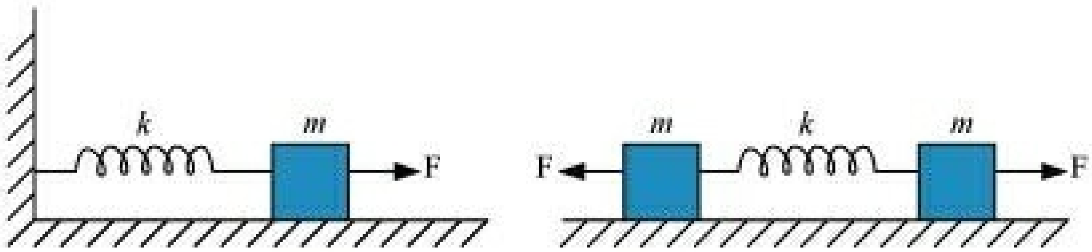
Angular velocity, ω = π rad/s

The motion of the particle can be plotted as shown in the following figure.



---

6. Figure 14.30 (a) shows a spring of force constant *k* clamped rigidly at one end and a mass *m* attached to its free end. A force *F* applied at the free end stretches the spring. Figure 14.30 (b) shows the same spring with both ends free and attached to a mass *m* at either end. Each end of the spring in Fig. 14.30(b) is stretched by the same force *F*.



(a) What is the maximum extension of the spring in the two cases?

(b) If the mass in Fig. (a) and the two masses in Fig. (b) are released, what is the period of oscillation in each case?

Ans.(a) For the one block system:

When a force F, is applied to the free end of the spring, an extension l, is produced. For the maximum extension, it can be written as:

$F = kl$

Where, k is the spring constant

Hence, the maximum extension produced in the spring, $l = \frac{F}{k}$

For the two block system:

The displacement (x) produced in this case is:

$$x = \frac{1}{2}$$

Net force, $F = +2\ kx\ 2k\frac{1}{2}$

$$\therefore l = \frac{F}{k}$$

(b) For the one block system:

For mass (m) of the block, force is written as:

$$F = ma = m\frac{d^2x}{dt^2}$$

Where, x is the displacement of the block in time t

$$\therefore m\frac{d^2x}{dt^2} = -kx$$

It is negative because the direction of elastic force is opposite to the direction of displacement.

$$\frac{d^2x}{dt^2} = -\left(\frac{k}{m}\right)x = -\omega^2 x$$

Where, $\omega^2 = \frac{k}{m}$

$$\omega = \sqrt{\frac{k}{m}}$$

Where,

ω is angular frequency of the oscillation

∴Time period of the oscillation, $T = \frac{2\pi}{\omega}$

$$= \frac{2\pi}{\sqrt{\frac{k}{m}}} = 2\pi\sqrt{\frac{m}{k}}$$

For the two block system:

$$F = m\frac{d^2x}{dr^2}$$

$$m\frac{d^2x}{dr^2} = -2kr$$

It is negative because the direction of elastic force is opposite to the direction of displacement.

$$\frac{d^2x}{dr^2} = -\left[\frac{2k}{m}\right]x = -\omega^2 x$$

Where,

Angular frequency, $\omega = \sqrt{\frac{2k}{m}}$

∴Time period, $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{2k}}$

---

7. One end of a U-tube containing mercury is connected to a suction pump and the other end to atmosphere. A small pressure difference is maintained between the two columns. Show that, when the suction pump is removed, the column of mercury in the U-tube executes simple harmonic motion.

Ans. Area of cross-section of the U-tube = A

Density of the mercury column = ρ

Acceleration due to gravity = g

Restoring force, F = Weight of the mercury column of a certain height

F = –(Volume × Density × g)

$F = -(A \times 2h \times \rho \times g) = -2\rho gh = -k \times$ Displacement in one of the arms (h)

Where,

$2h$ is the height of the mercury column in the two arms

k is a constant, given by $k = -\frac{F}{h} = 2A\rho g$

Time period, $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{2A\rho g}}$

Where,

m is the mass of the mercury column

Let l be the length of the total mercury in the U-tube.

Mass of mercury, m = Volume of mercury $\times$ Density of mercury

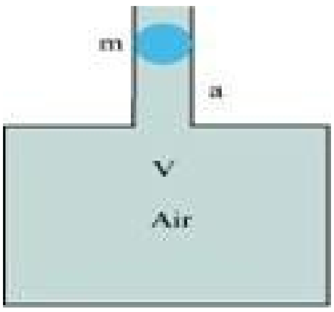
= $Al\rho$

$$\therefore T = 2\pi\sqrt{\frac{m}{2A\rho g}} = 2\pi\sqrt{\frac{l}{2g}}$$

Hence, the mercury column executes simple harmonic motion with time period $2\pi\sqrt{\frac{l}{2g}}$.

---

8. An air chamber of volume *V* has a neck area of cross section *a* into which a ball of mass *m* just fits and can move up and down without any friction (Fig.14.33). Show that when the ball is pressed down a little and released, it executes SHM. Obtain an expression for the time period of oscillations assuming pressure-volume variations of air to be isothermal [see Fig. 14.33].



Ans. Volume of the air chamber = V

Area of cross-section of the neck = a

Mass of the ball = m

The pressure inside the chamber is equal to the atmospheric pressure.

Let the ball be depressed by x units. As a result of this depression, there would be a decrease in the volume and an increase in the pressure inside the chamber.

Decrease in the volume of the air chamber, $\Delta V = ax$

Volumetric strain $= \frac{\text{Change in volume}}{\text{Original valume}}$

$$\Rightarrow \frac{\Delta V}{V} = \frac{ax}{V}$$

Bulk Modulus of air, $B = \frac{Stress}{Strain} = \frac{-p}{\frac{ax}{V}}$

In this case, stress is the increase in pressure. The negative sign indicates that pressure increases with a decrease in volume.

$$p = \frac{-Bax}{V}$$

The restoring force acting on the ball, $F = p \times a$

$$\frac{-Bax}{V} \cdot a$$

$$= \frac{-Ba^2 x}{V}$$

In simple harmonic motion, the equation for restoring force is:

$F = -kx$ ... (*ii*)

Where, k is the spring constant

Comparing equations (*i*) and (*ii*), we get:

$$= \frac{Ba^2}{V}$$

Time period, $T = 2\pi\sqrt{\frac{m}{k}}$

$$= 2\pi\sqrt{\frac{Vm}{Ba^2}}$$

9. You are riding in an automobile of mass 3000 kg. Assuming that you are examining the oscillation characteristics of its suspension system. The suspension sags 15 cm when the entire automobile is placed on it. Also, the amplitude of oscillation decreases by 50% during one complete oscillation. Estimate the values of (a) the spring constant *k* and (b) the damping constant *b* for the spring and shock absorber system of one wheel, assuming that each wheel supports 750 kg.

Ans.(a) Mass of the automobile, m = 3000 kg

Displacement in the suspension system, x = 15 cm = 0.15 m

There are 4 springs in parallel to the support of the mass of the automobile.

The equation for the restoring force for the system:

$F = -4kx = mg$

Where, k is the spring constant of the suspension system

Time period, $T = 2\pi\sqrt{\frac{m}{4k}}$

And $k = \frac{mg}{4x} = \frac{3000\times10}{4\times0.15} = 5000 = 5\times10^4\,N/m$

Spring constant, k = $5\times10^4\ N/m$

(b) Each wheel supports a mass, M = $\frac{3000}{7}$ = 750 kg

For damping factor b, the equation for displacement is written as:

$x = x_o e^{-bt/2M}$

The amplitude of oscillation decreases by 50%.

$\therefore x = \frac{x_0}{2}$

$$\frac{x_0}{2} = x_o e^{-bt/2M}$$

$$\log_e 2 = \frac{bt}{2M}$$

$$\therefore b = \frac{2M \log_e 2}{t}$$

Where,

Time period, $t = 2\pi\sqrt{\frac{m}{4k}} = 2\pi\sqrt{\frac{3000}{4 \times 5 \times 10^4}}$ = 0.7691 s

$$\therefore b = \frac{2 \times 750 \times 0.693}{0.7691}$$

= 1351.58 kg/s

Therefore, the damping constant of the spring is 1351.58 kg/s.

---

10. Show that for a particle in linear SHM the average kinetic energy over a period of oscillation equals the average potential energy over the same period.

Ans. The equation of displacement of a particle executing SHM at an instant *t* is given as:

$$x = A \sin \omega t$$

Where,

A= Amplitude of oscillation

ω = Angular frequency $= \sqrt{\frac{k}{M}}$

The velocity of the particle is:

$$v = \frac{dr}{dt} = A\omega \cos \omega t$$

The kinetic energy of the particle is:

$$E_k = \frac{1}{2}Mv^2 = \frac{1}{2}MA^2\omega^2 \cos^2 \omega t$$

The potential energy of the particle is:

$$E_p = \frac{1}{2}kx^2 = \frac{1}{2}M\omega^2 A^2 \sin^2 \omega t$$

For time period *T*, the average kinetic energy over a single cycle is given as:

$$(E_k)_{Avg} = \frac{1}{T}\int_0^T E_k \, dt$$

$$= \frac{1}{T}\int_0^T \frac{1}{2}MA^2\omega^2 \cos^2 \omega t \, dt$$

$$= \frac{1}{2T}MA^2\omega^2 \int_0^T \frac{(1+\cos 2\omega t)}{2} dt = \frac{1}{4T}MA^2\omega^2 \left[t + \frac{\sin 2\omega t}{2\omega}\right]$$

$$= \frac{1}{4T}MA^2\omega^2 (T)$$

$$= \frac{1}{4}MA^2\omega^2 \text{ ....(i)}$$

And, average potential energy over one cycle is given as:

$$(E_p)_{Avg} = \frac{1}{T}\int_0^T E_p \, dt$$

$$= \frac{1}{T}\int_0^T M\omega^2 A^2 \sin^2 \omega t \, dt$$

$$=\frac{1}{2T}M\omega^2A^2\int_0^T\frac{(1-\cos 2\omega)}{2}dt=\frac{1}{4T}M\omega^2A^2\left[t-\frac{\sin 2\omega t}{2\omega}\right]_0^T$$

$$=\frac{1}{4T}M\omega^2A^2(T) \text{.......(ii)}$$

It can be inferred from equations (*i*) and (*ii*) that the average kinetic energy for a given time period is equal to the average potential energy for the same time period.

---

11. A body describes simple harmonic motion with amplitude of 5 cm and a period of 0.2 s. Find the acceleration and velocity of the body when the displacement is (a) 5 cm, (b) 3 cm, (c) 0 cm.

Ans. Amplitude, *A*= 5 cm = 0.05 m

Time period, *T*= 0.2 s

(a) For displacement, *x*= 5 cm = 0.05 m

Acceleration is given by:

$a=-\omega^2x$

$$=-\left(\frac{2\pi}{T}\right)^2x=-\left(\frac{2\pi}{T}\right)^2\times 0.05$$

$$=-5\pi^2 m/s^2$$

Velocity is given by:

$v=\omega\sqrt{A^2-x^2}$

$$=\frac{2\pi}{T}\sqrt{(0.05)^2-(0.05)^2}$$

= 0

When the displacement of the body is 5 cm, its acceleration is

$-5\pi^2 m/s^2$ and velocity is 0.

(b) For displacement, *x*= 3 cm = 0.03 m

Acceleration is given by:

$a = -\omega^2 x$

$$= -\left(\frac{2\pi}{T}\right)^2 x = -\left(\frac{2\pi}{T}\right)^2 0.03$$

$$= -3\pi^2 m/s^2$$

Velocity is given by:

$v = \omega\sqrt{A^2 - x^2}$

$$= \frac{2\pi}{T}\sqrt{A^2 - x^2}$$

$$= \frac{2\pi}{T}\sqrt{(0.05)^2 - (0.03)^2} = \frac{2\pi}{0.2} \times 0.04$$

= 0.4 πm/s

When the displacement of the body is 3 cm, its acceleration is $-3\pi m/s^2$ and velocity is 0.4πm/s.

(c) For displacement, *x*= 0

Acceleration is given by:

$a = -\omega^2 x = 0$

Velocity is given by:

V= $= \omega\sqrt{A^2 - x^2}$

$$= \frac{2\pi}{T}\sqrt{A^2 - x^2}$$

$$= \frac{2\pi}{0.2}\sqrt{(0.05)^2 - 0}$$

$$= 0.5\pi m / s$$

When the displacement of the body is 0, its acceleration is 0 and velocity is 0.5 πm/s.